NAKIJK

Tentamen Regeltechniek TBKRT05E, 2 november 2011

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Exercise 1 (15 points of which 7 bonus) (conceptual exercise)

Assume you have to develop a maglev (magnetic levitation) train, such as the one currently operating in Shanghai at a speed of more than 400 km/hour given in the picture below. Recall that there are similarities with the magnetic levitated ball.

a). What is the main signal that can be controlled (the main control input)?

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the voltage (and consequently current) in the electromagnet of the rail

b). Mention a few outputs that need to be controlled.

5

height of the train
comfort (constant height, accelarations in vertical and horizontal direction)
speed

c). How can you determine transfer functions of a practical system like this?

5

Either by determining a black box model via system identification or by setting up a model based on physical laws, determine the parameters, linearize the system, put it in statespace form

then determine the Laplace transforms and set up the transfer function between inputs and outputs.

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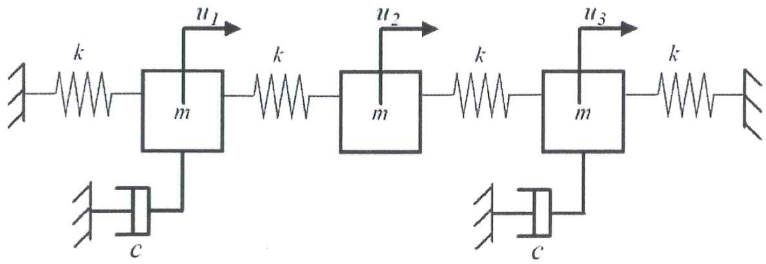
Excercise 2 (18 points)

Consider the mass-spring-damper system in the figure below.



The springs have spring constants k, the masses have mass m, and the damping constants are c. The external forces are given by u_1, u_2, and u_3.

a). How many states are minimally necessary for a state space description? Motivate your answer!

6

there are 3 masses and 4 springs:
7 states.

b). What is de Lagrangian of this system? Motivate your answer!

8

$$L(q,\dot q) = \tfrac{1}{2}\dot q_1^2 m + \tfrac{1}{2}\dot q_2^2 m + \tfrac{1}{2}\dot q_3^2 m - \tfrac{1}{2}k q_1^2 - \tfrac{1}{2}k(q_2-q_1)^2 - \tfrac{1}{2}k(q_3-q_2)^2 - \tfrac{1}{2}k q_3^2$$

q_1 position first mass, q_2 second mass, etc.
the wall has velocity zero.

c). What is the Rayleigh dissipation function of this system? Motivate your answer!

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$$D(\dot q) = \tfrac{1}{2}c\dot q_1^2 + \tfrac{1}{2}c\dot q_3^2$$

Excercise 3 (18 punten)

Consider an electrical circuit with a series connection between a voltage source, resistor, inductor and capacitor, where there is a nonlinear (diode like) resistor in parallel with the capacitor. The equations of motion are given by

$$\dot{x}_1 = -\frac{R}{L}x_1 - \frac{1}{L}x_2 + \frac{1}{L}u$$
$$\dot{x}_2 = \frac{1}{C}x_1 - \frac{1}{C}sin(x_2)$$

with $R, L, C > 0$, x_1 the current through the inductor, x_2 the voltage over the capacitor, and u the source voltage.

a). Determine the equilibrium points for $u = 0$. Motivate your answer!

6

$\dot{x}_1 = \dot{x}_2 = 0$ $\quad -Rx_1 = x_2$ and $x_1 = \sin x_2$

$\Rightarrow -R\sin x_2 = x_2 \iff$

$x_2 = 0 \pm k\pi, \; k = 0,1,2,$

3 3

b). Linearize the system around the point $u = 0, x_1 = 0, x_2 = 0$. Is this linear system stable, asymptotically stable or instable? Motivate your answer!

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$$f(x) = \begin{pmatrix} -\frac{R}{L}x_1 - \frac{1}{L}x_2 \\ \frac{1}{C}x_1 - \frac{1}{C}\sin x_2 \end{pmatrix}$$

$x = f(x) + g(x)u$

$$g(x) = \begin{pmatrix} \frac{1}{L} \\ 0 \end{pmatrix}$$

$$\frac{\partial f}{\partial x}(x)\Big|_{x=0} = \begin{pmatrix} -\frac{R}{L} & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{C}\cos x_2 \end{pmatrix}\Bigg|_{x=0} = \underbrace{\begin{pmatrix} -\frac{R}{L} & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{C} \end{pmatrix}}_{A}$$

3

$$g(0) = B = \begin{pmatrix} \frac{1}{L} \\ 0 \end{pmatrix}$$

eigenvalues(A): 3

$(\lambda + \frac{R}{L})(\lambda + \frac{1}{C}) + \frac{1}{LC} = 0$

$\Rightarrow \lambda^2 + (\frac{1}{C} + \frac{R}{L})\lambda + \frac{R+1}{LC} = 0$

$\dot{\bar{x}} = A\bar{x} + B\bar{u}$ with $\bar{x} = x$, $\bar{u} = u$

is the linearized system.

Z.O.Z.

$$\lambda = \frac{-\frac{1}{C} - \frac{R}{L} \pm \sqrt{\left(\frac{1}{C} + \frac{R}{L}\right)^2 - \frac{4(R+1)}{LC}}}{2}$$

$\Rightarrow Re(\lambda) < 0$, hence as. stable.

c). Linearize the system about the point $u = 0$, $x_1 = -\frac{\pi}{R}$, $x_2 = \pi$. Is the linear system stable, asyptotically stable or instable? Motivate your answer!

6

3 $A = \begin{pmatrix} -\frac{R}{L} & -\frac{1}{L} \\ \frac{1}{C} & +\frac{1}{C} \end{pmatrix}$ $B = \begin{pmatrix} \frac{1}{L} \\ 0 \end{pmatrix}$

3 $\det(\lambda I - A) = (\lambda + \frac{R}{L})(\lambda - \frac{1}{C}) + \frac{1}{LC} = 0$

$\Rightarrow \lambda^2 + (\frac{R}{L} - \frac{1}{C})\lambda - \frac{R-1}{LC} = 0$

$\Rightarrow \lambda = \frac{-\frac{R}{L} + \frac{1}{C}}{2} \pm \frac{1}{2}\sqrt{\left(\frac{R}{L} - \frac{1}{C}\right)^2 + \frac{4(R-1)}{LC}}$ $1\frac{1}{2}$

If ~~R<1 and~~

$\frac{1}{C} > \frac{R}{L}$, then instable,

If $\frac{1}{C} < \frac{R}{L}$, and $R > 1$, then instable

$1\frac{1}{2}$

If $\frac{1}{C} < \frac{R}{L}$, and $R < 1$ then as. stable

If $\frac{1}{C} < \frac{R}{L}$, and $R = 1$, then marg. stable

If $\frac{1}{C} = \frac{R}{L}$, then marg. stable.

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Excercise 4 (16 punten)

Consider the following continuous time state space system with input $u(t)$, output $y(t)$, and $a < 0$.

$$\dot{x}(t) = \begin{pmatrix} a & -1 \\ 1 & -2 \end{pmatrix} x(t) + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u(t)$$

$$y(t) = \begin{pmatrix} 1 & 0 \end{pmatrix} x(t)$$

a). Provide the transfer function of the system. Motivate your answer!

$$(sI-A)^{-1} = \begin{pmatrix} s-a & 1 \\ -1 & s+2 \end{pmatrix}^{-1} = \frac{1}{(s-a)(s+2)+1}\begin{pmatrix} s+2 & -1 \\ 1 & s-a \end{pmatrix}$$

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$$C(sI-A)B = \frac{s+2}{(s-a)(s+2)+1} = \frac{s+2}{s^2+(2-a)s-2a+1}$$

b). Consider the system for $a = 0, a = -1, a = -6, a = -10$. For these values of a, is dit systeem underdamped, overdamped, or critically damped? Motivate your answer!

$a=0$: $s^2+2s+1=0$ char. eq. $\omega_n = 1$, $\zeta = 1$ critically damped.

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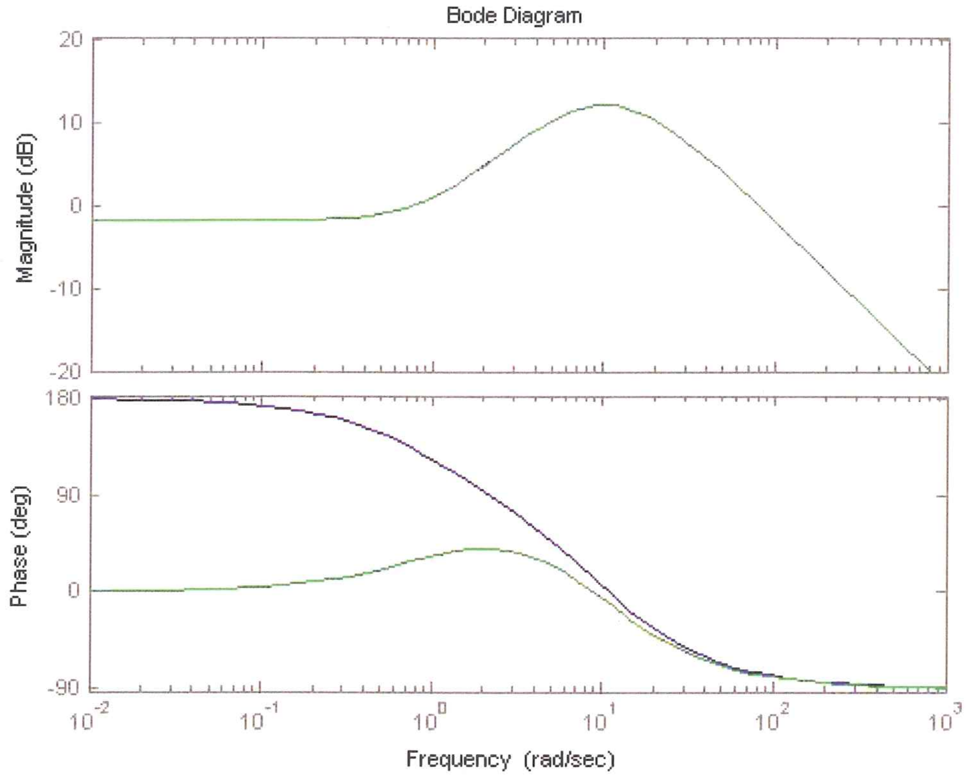
$a=-1$: $s^2+3s+3=0$ $\omega_n = \sqrt{3}$ $2\sqrt{3}\,\zeta = 3 \Rightarrow \zeta = \frac{1}{2}\sqrt{3} < 1$ 0,87 underdamped.

$a=-6$: $s^2+8s+13=0$ $\omega_n = \sqrt{13}$ $\zeta = \frac{8}{2\sqrt{13}} > 1$ overdamped 1,109

$a=-10$: $s^2+12s+21=0$ $\omega_n = \sqrt{21}$ $\zeta = \frac{12}{2\sqrt{21}} > 1$ overdamped. 1,309

Opgave 5 (20 punten)

Given the Bode plots of two stable open loop systems $G_1(s)$ (blue) and $G_2(s)$ (green) as in the plot below. The magnitude plot of the systems is equal, the phase plots differ.



a). Based upon the Bode plots, can you explain the difference in the transfer functions of these two systems?

10

Both systems have the same poles, and the blue one has a zero in the right half plane, where the green one has a zero of the same magnitude in the left half plane.

Z.O.Z.

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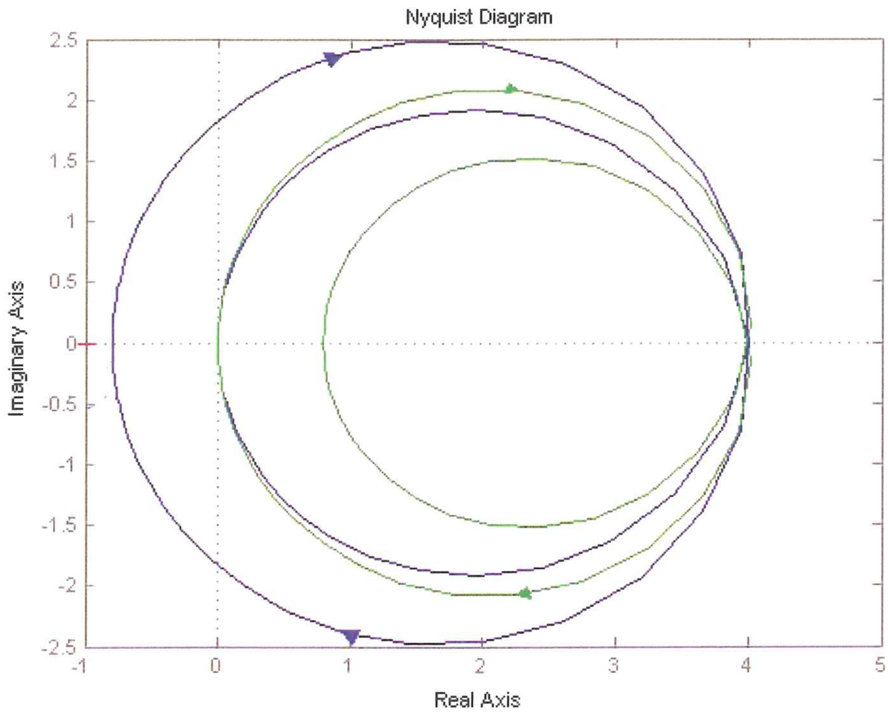
b). Consider the Nyquist plot of the same two stable systems $G_1(s)$ (blue) and $G_2(s)$ (green) as given below.



Is the closed loop system with loop transfer functions $L_1(s) = G_1(s)$ stable? The same question for $L_2(s) = G_2(s)$. Do there exist values for K_1 such that the closed loop with loop transfer function $L_1(s) = K_1G_1(s)$ is unstable? The same question for $L_2(s) = K_2G_2(s)$. If so, provide the values. Motivate your answer!

G_1 and G_2 are stable.

L_1, L_2 are both stable (with $K=1$)
since they are lying right of -1

10

For $K > \frac{1}{0.8}$, L_1 is unstable. (then -1 is encircled once)

L_2 is stable for all $K \geq 0$
(stays always in RHP)

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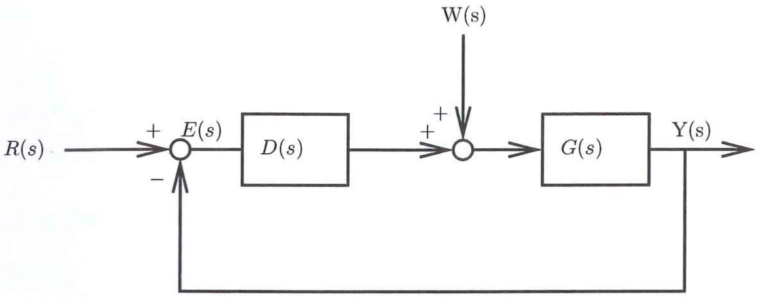
Opgave 6 (20 punten)

Given is the open loop system with transfer function: $G(s) = \dfrac{1}{(s+0.5)^2}$

The system has a reasonable steady state response for a step input signal if the loop is closed by unity feedback. However, the response is oscillatory and has high overshoot. We design a PD compensator

$$D(s) = K(1 + T_d s)$$



a). Consider the closed loop as in the block scheme above. What is the requirement for the controller if we want a steady state error of maximally 25% of the value of a constant step disturbance $W(s)$. Motivate your answer!

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$$e_{ss} = \lim_{s \to 0} s E(s) = \lim_{s \to 0} \frac{s G(s)}{1 + D(s) G(s)} \cdot \frac{A}{s}$$

absolute value since it is a relative error!

$$= \lim_{s \to 0} \frac{1}{(s+0.5)^2 + K + K T_d s} = \frac{A}{0.5^2 + K} \leq 0.25 A$$

with A the value of the constant step disturbance.

Hence $1 \leq 0.25(0.25 + K)$

$4 \leq 0.25 + K \Rightarrow K \geq 3.75$

Z.O.Z.

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Now consider the response to a reference signal. We aim at designing a controller such that the overshoot as response to a step reference signal is reduced to approximately 16 %, implying that we aim at a damping ratio of approximately $\zeta = 0.5$ for the closed loop, which on its turn is related to the desired phase margin.

b). What is the corresponding phase margin (PM) that we need? Motivate your answer!

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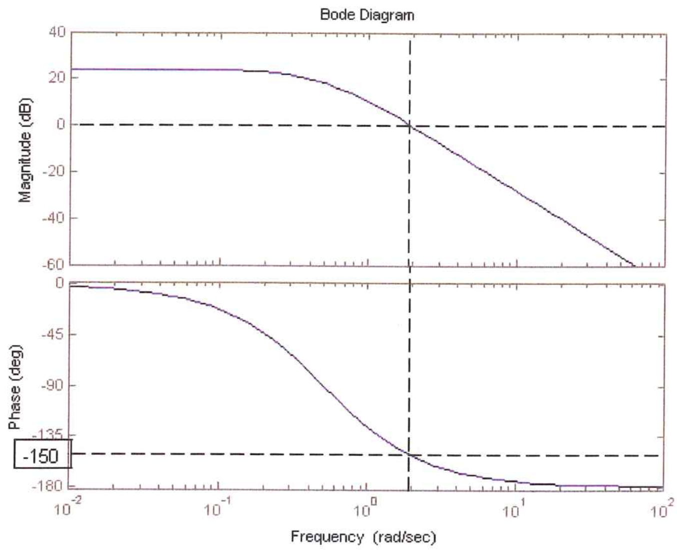
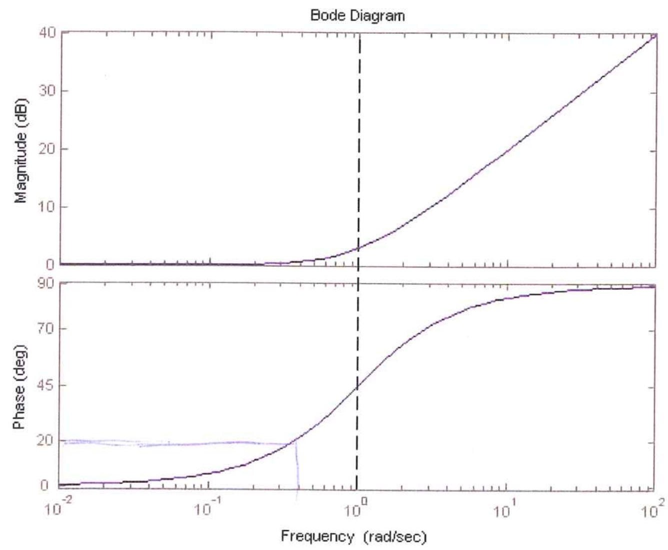
PM $\approx 50^\circ$, $\zeta \approx \frac{PM}{100}$

$PM = \arctan \frac{2\zeta}{\sqrt{\sqrt{1+4\zeta^4} - 2\zeta^2}}$

c). Consider the Bode plot of $KG(s)$, with K from excercise a). given as follows:



and recall that the Bode plot of the PD controller $1 + s$ is given by



Consider the cross over frequency of $KG(s)$ and determine how to pick T_d such that the cross over frequency stays approximately the same, and the PM is as desired. Motivate your answer!

$\omega_c = 2$, PM $= 30°$ for ~~voor~~ $KG(s)$

~~Dus~~ Hence minimaally $20°$ phase ~~toevoegen~~ addition. For $(1+s)$, this means at approx. $\omega = 0.5$, and ~~magn~~ magnitude of $1+s$ is then still approx. 1.

Hence, put $1/T_d$ at $\omega = 4$, then $T_d \approx 0.25$

6

d). Determine the natural frequency of the closed loop system. Furthermore, determine the settling time of the closed loop system. Motivate your answer!

$$1 + L(s) = 1 + \frac{4(1+0.25s)}{(s+0.5)^2} =$$

$\Leftrightarrow$ char. eq. $(s+0.5)^2 + 4 + s = 0$

$\Leftrightarrow s^2 + s + 0.25 + 4 + s = 0$

$\Rightarrow s^2 + 2s + 4.25 = 0$

$\omega_n = \sqrt{4.25}$, $\zeta = 0.5$

$t_s = \frac{4.6}{\zeta \omega_n} \approx$ 4,5 s. (for $\pm 1\%$)

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END EXAM